

A Hybrid Modeling and Optimal Control Framework for Layer-by-Layer Radiative Processing of Thick Sections

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Abstract— *This paper introduces a hybrid modeling and optimal control framework for a class of layer-by-layer manufacturing processes. Specifically, a stepped-concurrent layer-by-layer process is offered as a solution for overcoming the challenge of maintaining through-cure during thick-part fabrication using Ultraviolet (UV) radiation inputs that are subject to in-domain attenuation. The layering and curing sequence is modeled as a hybrid system, where the layering steps constitute discrete events on otherwise continuous curing kinetics and thermal processes. It is shown that the UV intensity as well as the inter-layer hold times can be selected optimally by posing an optimal control problem with the objective of minimizing the overall cure deviation in the thick multi-layer part. The necessary conditions for optimality are explicitly derived by adjoining the coupled PDE and ODE constraints of the process model. The potential benefit of the proposed optimization scheme is demonstrated considering simulations of a composite laminate curing process. It is found that, compared to traditional equal-interval layering, optimal layering time control gives significantly improved performance in terms of minimizing cure-level deviation, for comparable total energy usage. There is also some added benefit to optimizing the inter-layer UV input as well.*

Keywords: additive manufacturing, hybrid modeling of layer-by-layer manufacturing, optimal control of hybrid systems, radiative curing process

I. INTRODUCTION

A hybrid system is comprised of discrete and continuous dynamics. Perhaps the most prevalent types involve discrete switching of the continuous dynamics from one mode to another in response to internal conditions or external inputs [1] [1]. In many cases, a hybrid system is a natural model to describe the behavior of some dynamic systems and processes. Specific examples include: a steel annealing process, where an individual ingot passes through multiple furnaces with different operating conditions corresponding to certain quality requirements[2]; and chemical processes involving different phases of chemical treatment[3]. Furthermore, modeling a system as hybrid system has been pursued to simplify the analysis of complex physical problems. For example, heavily nonlinear problems may be handled by via approximations with a number of linear or affine systems in different operating regimes/modes[4].

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Application examples of the latter approach include obstacle avoidance[5], traffic control[6] and biological systems[7].

In this paper, we introduce the hybrid modeling and optimal control framework for a class of layer-by-layer additive manufacturing processes. In such processes, the addition of each layer is a naturally discrete event that may change the evolution of the underlying physical phenomena (thermal or otherwise) in the part build-up process. As a specific application, we consider the layer-by-layer manufacturing of parts via the ultraviolet (UV) radiative curing of resins and laminates.

UV curing is widely used for photopolymerization of thin film such as paints and coatings[8]. The potential of UV curing for making complex parts has also been demonstrated via the layer-by-layer stereolithography process [9]. More broadly, UV curing is gaining substantial interest for substituting alternative thermal processes in curing thick parts such as composite laminates [10]. This is due to its advantages of accelerated processing time, higher-energy efficiency, less environmental pollution, reduced space usage, and better controllability[8, 11]. However, some challenges remain. One of the key challenges is the differing material shrinkage and thermal stresses between layers due to cure level and temperature gradients across the depth of the part. The combination of these often leads to undesirable distortions in the end product [12].

To overcome these challenges, in our previous work[13], we proposed a stepped-concurrent layering and curing(SCC) processes, where new layers are added before previous ones cure completely in such a way that there is an effective reduction of cure level deviation and thermal stress in all layers. In SCC, the successive addition of each layer changes both the spatial domains and the initial conditions of the physical processes. In fact, SCC can be framed as a multi-mode hybrid system with a pre-defined mode sequence and a growing spatial domain. In[13], we motivated that there are optimal inter-layer hold times for SCC and developed a systematic optimization scheme to compute this optimal hold times which are treated as the control inputs. In the current paper, we build on that work as follows: 1) we consider the UV input intensity as an explicit control variable that also needs to be optimized in addition to the inter-layer hold times; and 2) we broaden the hybrid modeling framework by considering the complete coupled PDE-ODE (partial and ordinary differential equations) UV curing process model for deriving the optimality conditions, unlike the augmented ODEs treated in our previous work.

In the literature, there are few works on the optimal control of hybrid systems whose modes involve PDE

models[6] while a lot of work exists for those involving solely of ODEs[1, 14]. In principle, for systems involving solely of PDEs or coupled ODEs and PDEs, one can derive first-order optimality conditions considering either discretize-then-optimize or optimize-then-discretize approaches via adjoint-based techniques [15]. In the current paper, we derive the first-order necessary conditions for optimality by directly adjoining the coupled PDE-ODE constraints within the hybrid optimal control framework for the layer-by-layer curing process. Our ideas loosely build on[16], which treated control of a single, non-hybrid PDE-ODE system. We set the objective of achieving minimal cure level deviations at the end of the curing process as the objective. The adjoint system and optimality conditions are then solved to compute the optimal process control variables: UV input intensity and inter-layer hold times. We illustrate the effectiveness of the proposed scheme by simulating a fiberglass composite curing process under different control scenarios.

The remainder of the paper is organized as follows. Section II gives a generalized 1D model for a UV curing process and the hybrid modeling set up for the layer-by-layer curing process. Section III provides the optimality conditions. Section IV offers demonstrative numerical simulation results and discussions. Section V gives the conclusions of the work.

II. PROBLEM STATEMENT

A. 1D UV Curing Process Model

Consider the schematic of the 1D UV curing processes for a single resin layer or thick section shown in Fig. 1. The curing process involves cure kinetics, heat generation by the exothermic cure reaction and heat transfer via conduction and convection. There is also attenuation of UV intensity across the layer in the z -direction according to Beer Lambert's Law. Other modeling consideration can be referred from [13, 17]. The following coupled PDE-ODE systems along with the boundary and initial conditions, summarize the process model:

$$\rho c_p \frac{\partial T(z,t)}{\partial t} = \frac{\partial}{\partial z} \left(k_z \frac{\partial T(z,t)}{\partial z} \right) + v_r \Delta H_r \rho_r \frac{d\alpha(z,t)}{dt} \quad (1a)$$

$$-k_z \frac{\partial T(0,t)}{\partial z} + \vartheta I_0 = h(T(0,t) - T_\infty) \quad (1b)$$

$$\frac{\partial T(l,t)}{\partial z} = 0 \quad (1c)$$

$$T(z,0) = T_0(z) \quad (1d)$$

$$\frac{d\alpha(z,t)}{dt} = I_0^p K(z,T) \alpha^m(z,t) (1 - \alpha(z,t))^n \quad (1e)$$

$$K(z,T) = \varphi S^q \exp(-\lambda p z) \exp\left(\frac{-E}{RT_{abs}(z,t)}\right) \quad (1f)$$

$$\alpha(z,0) = \alpha_0(z) \quad (1g)$$

where ρ and c_p are the density and specific heat capacity of the composite laminate, respectively; k_z is the thermal conductivity of the laminate in the z -direction; $T(z,t)$ is temperature distribution at depth z and time t ; v_r is volumetric fraction of resin in the composite matrix; ρ_r is density of resin; and ΔH_r is polymerization enthalpy of resin conversion; E is activation energy, S is photoinitiator concentration, φ is pre-exponential factor of rate constant; R is gas constant; I_0 is UV input intensity at the surface; $T_{abs}(z,t)$ is absolute temperature in Kelvin; $\alpha(z,t)$ is cure

level/state distribution; m & n are reaction orders; p & q are constant exponents; λ is the absorption coefficient in the resin plus fiber; ϑ is absorptivity constant of the UV radiation at the boundary; h is convective heat transfer at the top boundary; l is the thickness of a single layer, and T_∞ is constant ambient temperature; and $d\alpha(z,t)/dt$ is the rate of cure conversion (rate of polymerization).

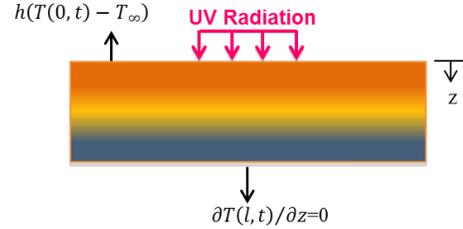


Figure 1: Schematic of a UV Curing Process

B. Formulation of a Layer-by-Layer UV Curing Process as a Hybrid System

As already mentioned, as a new layer is introduced for curing, the spatial domain, initial conditions and boundary conditions change, resulting in a different process “mode”. This mode switch represents a discrete event and the switching or layering instants are decided externally to the curing process. This hybrid system view of the layer-by-layer curing process is depicted schematically in Fig. 2. In the following, a “mode” represents the state dynamics before the addition of a new layer. The first mode (Mode 1) has only 1 layer, and all other modes have more, in increasing numbers as shown. The mode switching times are denoted by τ_1 through τ_N . In this hybrid system view, the switching/layering times as well as the UV radiation input at the top layer are control variables that can be manipulated for a desired effect, in this case, for minimization of cure level deviations in a multi-layer part.

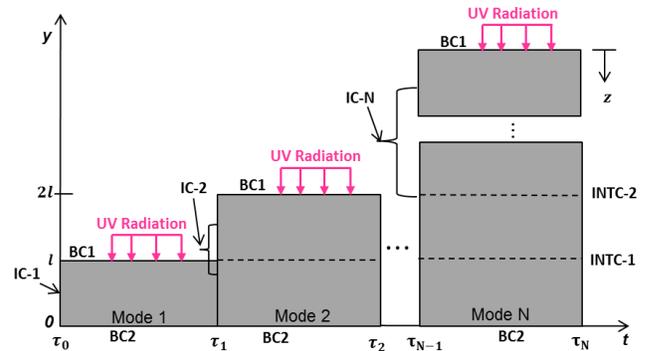


Figure 2: A hybrid system formulation of the layer-by-layer curing process

For this hybrid system realization of the layer-by-layer curing process, the following observations and assumptions can be made: 1) At each mode switch (layer addition), from mode i to the next mode $i + 1$, the spatial domain grows and the initial conditions (IC) change from $IC-i$ to $IC-i+1$. 2) The process dynamics in mode i can be treated as a single coupled PDE-ODE system with introduction of an interface condition (INTC) that captures the heat transfer between the fresh layer and the layers already in the curing process. The INTC for the curing process is defined in (4) below. 3) The

boundary condition (BC) of top convective (BC1) and bottom insulation (BC2) are kept the same for all modes. 4) Since one can only add layers, the order of mode switching is fixed, sequential and known; 5) All of the mode switching times included in the ordered vector $[\tau_1, \dots, \tau_N]^T$ can be selected independently. Note that, in Fig. 2, the y-axis indicates the increasing spatial domain with layer addition from bottom to top while the z-axis indicates the direction of UV attenuation. The UV source is at the top.

Denoting the thickness of the part after the i^{th} layer is added by il and introducing a coordinate transformation $y = il - z$ between the global y-axis and the local z-axis, and introducing notations $T_{it}(y, t), T_{iy}(y, t), T_{iyy}(y, t)$ and $\alpha_{it}(y, t)$ for $\partial T_i(y, t)/\partial t, \partial T_i(y, t)/\partial y, \partial^2 T_i(y, t)/\partial y^2$ and $\partial \alpha_i(y, t)/\partial t$, respectively, the state evolution for mode i in the time interval, $t \in [\tau_{i-1}, \tau_i]$ takes the form:

$$T_{it}(y, t) = aT_{iyy}(y, t) + b(y)f_i(T_i(y, t), \alpha_i(y, t), u_i(t)) \quad \text{on } \Omega_{it} \quad (2a)$$

$$T_{iy}(il, t) + eu_i(t) = c(T(il, t) - T_\infty) \quad \text{on } \Gamma_{i1} \quad (2b)$$

$$T_{iy}(il, t) = 0 \quad \text{on } \Gamma_{i2} \quad (2c)$$

$$\alpha_{it}(y, t) = d(y)f_i(T_i(y, t), \alpha_i(y, t), u_i(t)) \quad \text{on } \Omega_{it} \quad (2d)$$

where both the temperature state $T_i(y, t)$ and cure state $\alpha_i(y, t)$ evolve in the spatio-temporal domain defined by $\Omega_{it} = [0, il] \times [\tau_{i-1}, \tau_i]$. $0 \leq \tau_0 < \tau_1 < \dots < \tau_N < \infty$. The boundary conditions are also defined on $\Gamma_{i1} = \{il\} \times [\tau_{i-1}, \tau_i]$, and $\Gamma_{i2} = \{0\} \times [\tau_{i-1}, \tau_i]$. $u_i(t) \in \mathfrak{R}$ is a scalar continuous time UV input intensity. The nonlinear function f_i is:

$$f_i(T_i(y, t), \alpha_i(y, t), u_i(t)) = \varphi u_i^p(t) \exp\left(\frac{-E}{RT_{iabs}(y, t)}\right) \alpha_i^m(y, t) (1 - \alpha_i(y, t))^n \quad \text{on } \Omega_{it} \quad (3)$$

and $(y) = S^q \exp(-\lambda p(il - y))$, $b(y) = (v_r \Delta H_r \rho_r / \rho c_p) d(y)$, $a = k_z / \rho c_p$, $c = h / k_z$, and $e = \vartheta / k_z$.

For two or more layers, at the addition of a new layer, the interface conditions (INTC) at $i = 1, 2, \dots, N - 1$ are:

$$[k_z T_{iy}(il, t)]_{\text{new layer}} = [k_z T_{iy}(il, t)]_{\text{previous layer}} \quad (4a)$$

$$[T_i(il, t)]_{\text{new layer}} = [T_i(il, t)]_{\text{previous layer}} \quad (4b)$$

At each switching time $\tau_i, i = 1, 2, \dots, N - 1$, the transition to the new mode defines new initial conditions for the next mode. This is described compactly by:

$$T_{i+1}(y, \tau_i^+) = F_i(T_i(y, \tau_i^-), T_0(y)) \quad (5a)$$

$$\alpha_{i+1}(y, \tau_i^+) = G_i(\alpha_i(y, \tau_i^-), \alpha_0(y)) \quad (5b)$$

where, $T_i(y, \tau_i^-)$ and $T_{i+1}(y, \tau_i^+)$ are the left hand and right hand limit values of the temperature state in mode i and mode $i + 1$, respectively, at the switching time τ_i . $F_i: \Omega_i \rightarrow \Omega_{i+1}$ is the mode transition operator for the temperature state at switching time τ_i : $\Omega_i \in [0, il]$. Since both states coexist in the spatial domain in all modes, similar definitions hold for the cure state (5b) as well.

To give a particular example of the mode transition operator for this application, we enforce continuity in the temperature state by taking the average temperature at the interface of new layer and the layer in the curing process at

switching time τ_i . The cure state at the interface is taken as that of the cure state already in the curing process, because cure conversion is an irreversible process. For all other locations in the domain away from the interfaces that were already being cured (all previous layers), the initial values of the temperature and cure states in the new mode take their values from the end of the previous mode. Of course, the initial value of all state elements corresponding locations in the new layer will take on ambient conditions.

Temperature state mode transition:

$$F_i(T_i(y, \tau_i^-), T_0(y)) = \begin{cases} T_i(y, \tau_i^-), & 0 \leq y < il \\ \frac{1}{2}(T_i(y, \tau_i^-) + T_0(y)), & y = il \\ T_0(y), & il < y \leq (i+1)l \end{cases} \quad (6a)$$

Cure state mode transition:

$$G_i(\alpha_i(y, \tau_i^-), \alpha_0(y)) = \begin{cases} \alpha_i(y, \tau_i^-), & 0 \leq y \leq il \\ \alpha_0(y), & il < y \leq (i+1)l \end{cases} \quad (6b)$$

Equations (2-6) complete the hybrid formulation for the layer-by-layer UV curing process.

III. OPTIMAL CONTROL OF THE HYBRID SYSTEM

For hybrid system described by (2-6), the optimal control problem can be posed as one of finding the optimal continuous input $u_i(t)$ and switching time vector $[\tau_1, \dots, \tau_N]^T$ that minimize a cost function of the following form:

$$J = \sum_{i=1}^N \int_{\tau_{i-1}}^{\tau_i} \int_{\Omega_i} L_i(T_i(y, t), \alpha_i(y, t), u_i(t), T_d(y), \alpha_d(y)) dy dt + \int_{\Omega_N} g(T_N(y, \tau_N), \alpha_N(y, \tau_N)) dy + \sum_{i=1}^{N-1} \gamma_i(\tau_i^-) \quad (7)$$

where L_i is a continuous functional that defines the relative distance between the actual and desired temperature state $T_d(y)$ and cure state $(\alpha_d(y))$, as well as the energy utilization in mode i . $\gamma_i(\tau_i^-)$ is the cost associated with switching at τ_i , and g is a terminal cost at final time τ_N . The initial time τ_0 and state $T(y, \tau_0)$ & $\alpha(y, \tau_0)$ are assumed fixed, while the final time τ_N and state $T(y, \tau_N)$ & $\alpha(y, \tau_N)$ are free to be optimized.

A. Optimality Conditions

In order to derive the necessary conditions for optimality, we first adjoin the dynamic constraint (2) and the transition constraint (5) to the cost function (7) using Lagrange multipliers $\bar{p}_i(y, t)$ for temperature dynamics, $\bar{q}_i(y, t)$ for cure dynamics, $\mu_i(y, \tau_i^-)$ for temperature transition constraint, and $\eta_i(y, \tau_i^-)$ for cure level transition constraint. The necessary conditions for first-order optimality are stated as follows. The derivation is given in the extended version of the paper[18].

Necessary conditions: Modeling the layer-by-layer curing process as a hybrid system of the form (2-6) and neglecting the switching cost, an extremum to the cost defined in (7) can be achieved by choosing control variables u_i and $\tau_i, i = 1, 2, \dots, N$ to satisfy the following conditions:

a) For $t \in [\tau_{i-1}, \tau_i]$, the following adjoint equation holds

$$\bar{p}_{it}(y, t) = -a\bar{p}_{iyy}(y, t) - \{b(y)\bar{p}_i(y, t) + d(y)\bar{q}_i(y, t)\} * \frac{\partial f_i(T_i(y, t), \alpha_i(y, t), u_i(t))}{\partial T_i} - \frac{\partial L_i(T_i(y, t), \alpha_i(y, t), u_i(t), T_d(y), \alpha_d(y))}{\partial T_i} \quad \text{on } \Omega_{it} \quad (8a)$$

$$\bar{p}_{iy}(il, t) = c\bar{p}_i(il, t) \quad \text{on } \Gamma_{i1} \quad (8b)$$

$$\bar{p}_{iy}(0, t) = 0 \quad \text{on } \Gamma_{i2} \quad (8c)$$

$$\bar{q}_{it}(y, t) = -\{b(y)\bar{p}_i(y, t) + d(y)\bar{q}_i(y, t)\} \frac{\partial f_i(T_i(y, t), \alpha_i(y, t), u_i(t))}{\partial \alpha_i} - \frac{\partial L_i(T_i(y, t), \alpha_i(y, t), u_i(t), T_d(y), \alpha_d(y))}{\partial \alpha_i} \quad \text{on } \Omega_{i\tau} \quad (8d)$$

b) At $t = \tau_N$, the adjoint variable $\bar{p}(y, t)$ and $\bar{q}(y, t)$ should satisfy

$$\bar{p}_N(y, \tau_N^-) = \frac{\partial g(T_N(y, \tau_N^-), \alpha_N(y, \tau_N^-))}{\partial T_N} \quad (9a)$$

$$\bar{q}_N(y, \tau_N^-) = \frac{\partial g(T_N(y, \tau_N^-), \alpha_N(y, \tau_N^-))}{\partial \alpha_N} \quad (9b)$$

c) At any time $t = \tau_i$, $i = 1, 2, \dots, N-1$, we have

$$\bar{p}_i(y, \tau_i^-) = \frac{\partial F_i^T(T_i(y, \tau_i^-), T_0(y))}{\partial T_i} \bar{p}_{i+1}(y, \tau_i^+) \quad (10a)$$

$$\bar{q}_i(y, \tau_i^-) = \frac{\partial G_i^T(\alpha_i(y, \tau_i^-), \alpha_0(y))}{\partial \alpha_i} \bar{q}_{i+1}(y, \tau_i^+) \quad (10b)$$

d) For $t \in [\tau_{i-1}, \tau_i]$, the following optimality conditions for u_i in (11a), τ_i , $i = 1, 2, \dots, N-1$ in (11b) and τ_N in (11c) should hold

$$\int_{\Omega_i} \left\{ [\bar{p}_i(y, t)b(y) + \bar{q}_i(y, t)d(y)] \frac{\partial f_i(T_i(y, t), \alpha_i(y, t), u_i(t))}{\partial u_i} \right\} dy + \frac{\partial L_i(T_i(y, t), \alpha_i(y, t), u_i(t), T_d(y), \alpha_d(y))}{\partial u_i} - ae\bar{p}_i(il, t) = 0 \quad (11a)$$

$$H_i(\tau_i^-) - H_{i+1}(\tau_i^+) = 0 \quad (11b)$$

$$H_N(\tau_N^-) = 0 \quad (11c)$$

where,

$$H_i(\tau_i^-) = \int_{\Omega_i} [L_i(T_i(y, \tau_i^-), \alpha_i(y, \tau_i^-), u_i(t), T_d(y), \alpha_d(y)) + \{b(y)\bar{p}_i(y, \tau_i^-) + d(y)\bar{q}_i(y, \tau_i^-)\} f_i(T_i(y, \tau_i^-), \alpha_i(y, \tau_i^-), u_i(\tau_i^-)) + a\bar{p}_i(y, \tau_i^-) T_{iyy}(y, \tau_i^-)] dy \quad (12)$$

Remark I: The continuous input $u_i(t)$ can be mode-wise constant, i.e, only a constant, yet optimizable input is applied in mode i . The condition in (11a) may be modified for mode-wise constant input as follows:

$$\int_{\tau_{i-1}}^{\tau_i} \left[\int_{\Omega_i} \left\{ [\bar{p}_i(y, t)b(y) + \bar{q}_i(y, t)d(y)] \frac{\partial f_i(T_i(y, t), \alpha_i(y, t), u_i(t))}{\partial u_i} \right\} dy + \frac{\partial L_i(T_i(y, t), \alpha_i(y, t), u_i(t), T_d(y), \alpha_d(y))}{\partial u_i} - ae\bar{p}_i(il, t) \right] dt = 0 \quad (13)$$

Remark II: To solve the optimality conditions given by (8-11), at least the first-derivative (Jacobian) of functions g , f_i , L_i , F_i & G_i must exist and be continuous. In the present curing problem where the cure kinetics is modeled by the phenomenological model (2d), $\partial f_i / \partial \alpha_i$ becomes singular for α close to zero and reaction order $m < 1$. To compute the Jacobian, one needs to replace the cure kinetic model (2d) with a non-singular phenomenological model; for example from[19]:

$$\frac{\partial \alpha_i(y, t)}{\partial t} = d(y)u^p(t)[K_1(T_i) + K_2(T_i) \alpha_i(y, t)](1 - \alpha_i(y, t))(B - \alpha_i(y, t)) \quad (14a)$$

where

$$K_1(T_i) = A_1 \exp\left(\frac{-E_1}{RT_{iabs}(y, t)}\right) \quad (14b)$$

$$K_2(T_i) = A_2 \exp\left(\frac{-E_2}{RT_{iabs}(y, t)}\right) \quad (14c)$$

For simulation purposes in section IV, we shall use the cure kinetic model in (2d) as a process model and use the preferred model (14) for solving the optimality conditions. We identify the constant parameters (A_1, A_2, E_1, E_2 & B) in (14) to closely approximate (2d).

B. Computation Algorithm

Based on the above necessary conditions for optimality, a standard steepest descent algorithm can be applied to solve for the optimal layering time vector $[\tau_1, \dots, \tau_N]^T$ and mode-wise constant input. For varying input $u_i(t)$, the standard algorithm needs to be modified in order to overcome difficulties with changing switching times[20]. For example, there is a possibility for input chattering at switching instants while differentiating the input for modification. This is one of the challenges with hybrid formulation and needs further investigation. Here, we restrict the discussion to the mode-wise constant input case and use the steepest descent algorithm given in[13] with Armijo's step size[21] to compute the control variables: u_i & $\tau_i, i = 1, \dots, N$.

IV. RESULTS AND DISCUSSION

In this section, we present simulation results to demonstrate the proposed scheme by applying it to composite laminate fabrication (fiberglass with unsaturated polyester resin) via layer-by-layer UV curing process. Here, we are interested in achieving near through cure in all layers at the end of the curing process by optimizing the switching time sequence (layering time) and UV irradiance input simultaneously. The cost function in (7) retains only the terminal cost of the form: $J = 1/2 \int_0^{M_i} \{\alpha(y, \tau_N^-) - \alpha_d(y)\}^2 dy$ with no transition cost, assuming instantaneous layering operations with constant cost. With these considerations, we can choose $L_i = 0$, and the optimality conditions (8-11) can be significantly simplified.

For the simulation study, the associated thermal, chemical and material constants for photopolymerization of unsaturated polyester resin are extracted from published work [22]. For the fiberglass, E-glass thermal properties such as thermal conductivity ($k_z = 0.012W/cm \cdot ^\circ C$), specific heat ($c_p = 0.8J/g \cdot ^\circ C$), and density ($\rho = 2.55g/cm^3$) are used. The resin volume fraction is assumed to be 60% for computing the average thermal properties of the composite laminate.

The simulation considers the UV curing model (2) to generate the temperature and cure state distributions in all layers. The modified cure kinetics model in (14) is used to compute the relevant Jacobians. For the process simulation and implementation of the optimization algorithm, a 10-node spatial discretization is adopted for each layer to convert the temperature PDE and corresponding adjoint PDE to a set of ODEs in time. Then, the temperature states are computed forward in time from 0 to τ_N while the adjoint variables are computed backward in time from τ_N to 0. A total of 10 layers with a thickness of 1 mm each are considered. The desired/target final cure level is specified to be 90% across all layers.

We illustrate the advantages of the proposed scheme with three cases with some optimality and one non-optimal case:

Case1: Optimized layering time control combined with optimal mode-wise constant input.

Case 2: Optimized layering time control combined with a constant UV input throughout the process.

Case 3: Equal-interval layering time combined with optimal mode-wise constant input.

Case 4: A non-optimal approach with equal-interval layering time and constant UV input.

For the first two optimal cases, an equal-interval layering time of 80s and a constant UV input of $65\text{mW}/\text{cm}^2$ are used as baselines to initiate the optimization. In Case 2, the constant UV input is kept at the baseline of $65\text{mW}/\text{cm}^2$. For Case 3, the equal-interval is determined by setting the overall curing time to be the same as the result for the optimal case, Case 1. For the non-optimal case, Case 4, the total energy and the length of overall curing time are selected to be the same as those achieved with the optimal case, Case 1. For the three optimal cases, the optimization is executed until the desired performance is achieved. The results are given in Figs. 3-5 below.

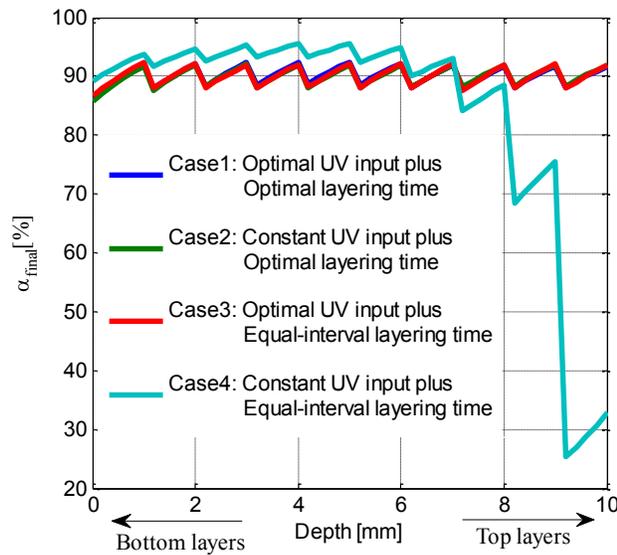


Figure 3: Achieved final cure level profile with optimal and non-optimal control

Figure 3 shows the final cure level distribution achieved for the final 10-layer part. For the non-optimal case (Case 4) of equal-interval layering time, the achieved cure level exceeds the target in the layers at the bottom while dropping dramatically for the last 2-3 top layers. The final cure level distribution achieved with the allocated time and energy is not acceptable. All three proposed model-based optimization results (Case 1, Case 2 and Case 3) offer a much better uniformity in the final of cure level with less than 5% overall cure level deviation across the part. However, there are some differences between the optimal cases in terms of energy usage and overall curing times as shown in Figs. 4 and 5, respectively.

Figure 4 shows that the optimal inter-layer hold times show the same trend in Case 1 and Case 2. It first decreases as one adds layers from the bottom up and then increases for the top layer. The larger hold times computed for the early bottom layers can be explained by the anticipation via optimization of the attenuation of UV radiation in the bottom

layers as new layers are add on. The largest hold time for the last and top layer can be explained by the need for bringing the cure level there from zero to the desired level quickly while the cure level continues to build in the lower layers with attenuated UV radiation. For Case 3, the constant time interval for layer addition is pre-specified.

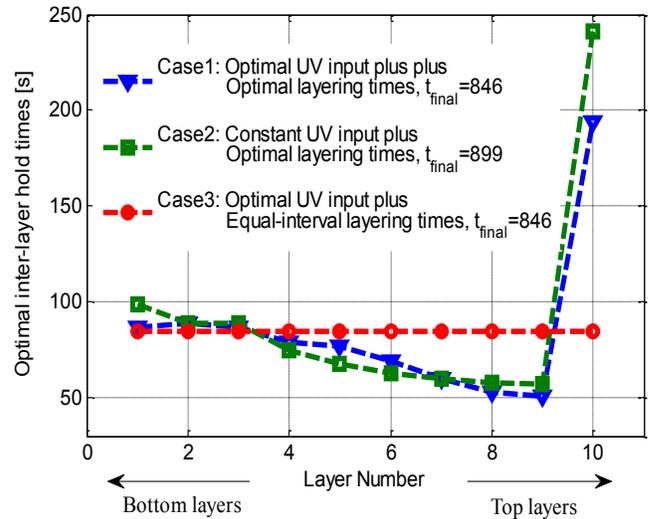


Figure 4: Optimized inter-layer hold times for the three optimal cases

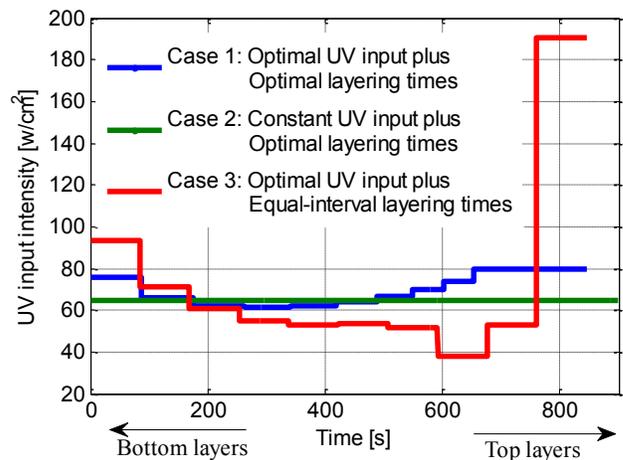


Figure 5: Control input of UV input intensity for three optimal cases

Figure 5 shows that, for optimal Case 1, the UV input first decreases as layers are added on and becomes smallest in magnitude in the middle and then picks up as the final top layers are added on. This trend is also explained by UV attenuation in the layers. The combined optimal control of mode-wise constant input and inter-layer hold times (Case 1) slightly reduces the overall curing time as compared to optimal inter-layer hold times plus constant UV input (Case 2) with comparable overall energy usage (see Table 1). The case of optimal UV input plus equal-interval layering time (Case 3) tracks the desired cure distribution with close performance to that of Case 1 and Case 2, but with relatively higher total energy. Furthermore, the highest UV input demand for the last top layers in Case 3 may not be practical when considering UV source capabilities.

Table 1: Final curing time and total energy consumption for considered cases

Considered cases	Final curing time in (s)	Total energy consumption in (KJ/cm ²)
Case 1	846	58.8
Case 2	899	58.4
Case 3	846	60.8
Case 4	846	58.8

V. CONCLUSIONS

In this paper, a hybrid system formulation and optimal control approach has been outlined for a layer-by-layer manufacturing process. Considering a specific layer-by-layer UV curing process, we derived the optimality conditions that offer a systematic computation of the optimal control variables (inter-layer hold times and UV input intensity) that minimize the deviation of cure level in a multi-layer thick part. The 1D UV curing process model, which is a coupled PDE-ODE model describing the thermal and curing kinetics, is used to construct the hybrid system interpretation of the layering and curing process by defining suitable inter-layer interface conditions for the layer-by-layer process. Then, the first-order optimality conditions are given. We demonstrated the benefit of the presented scheme through simulations of composite laminate fabrication process, where three optimality cases are compared with a non-optimal case. It is observed that, for the process model simulated, optimizing the layering time and the mode-wise UV input gives the best results in terms of minimizing cure level deviations with least total energy and time. The most significant impact comes from optimizing the layering times.

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